Question 1, 2016
AP Calculus BC
Test

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

a. $R'(2) = \frac{950-1190}{3-1} = \frac{-240}{2} = \boxed{-120 \text{ liters/hr}^2}$ unite: $\frac{\text{liters/hr}}{hr} = \frac{liters}{hr^2}$

answer

1	3
1190	950

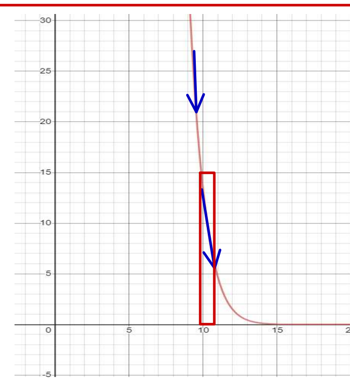
2 is missing in the table, so use 3,1, 950 and 1190 to approximate it.

b. $\int_0^8 R(t)\,dt \approx 1340\cdot1 + 1190\cdot2 + 950\cdot3 + 740\cdot2 = \boxed{8050 \text{ liters}}$

answer

It's an overestimate since R(t) is a decreasing function.

You see this fact in the graph on the right, as ong as you're looking at it from left to right. The red rectangle shows more area than there actually is under the graph.



c. total amount of water at the end of 8 hours:
Total water=initial amount + water in +water out
"out" indicates a negative sign here

$$\text{Total Amount} = \underbrace{50000+}_{\text{initial at } t=0} \underbrace{\int_0^8 2000\,e^{-t^2/20}\,dt}_{\text{water in}} + \underbrace{\int_0^8 R(t)\,dt}_{\text{water out estimate from part b}}$$

$$= 50000 + 7836.19 \underbrace{-\,8050}_{\text{subtract since water is flowing out}} = \boxed{49786\,Liters} \quad answer$$

d. Total rate of change=Rate of Inflow+Rate of Outflow=W(t)-R(t)
R(t) is differential, so it's continous. W(t) is continous based on the kind of function it is.
So T(t)=W(t)-R(t) is continous.
T(0)= $W(0) - R(0) = 2000\,e^{-0^2/20} - 1340 = 2000 - 1340 > 0$
$T(8) = W(8) - R(8) = 2000\,e^{-8^2/20} - 700 < 0$

Since the signs change, and T(t) is continous, (since it's a combination of continous functions), by the Intermediate Value Theorem there is a t=c such that T(c)=0 . At this value of t=c, the rates of inflow and outflow are the same.