question 3 from the 2018 free response section

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

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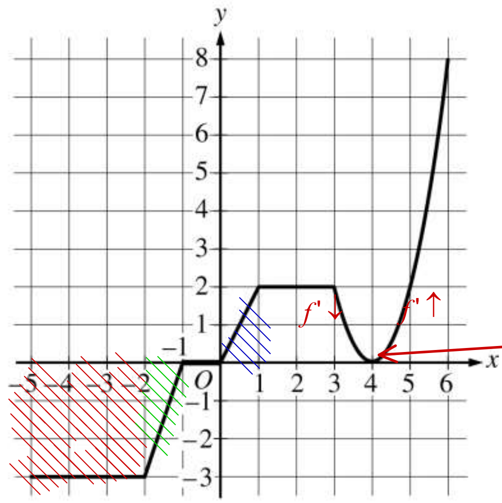
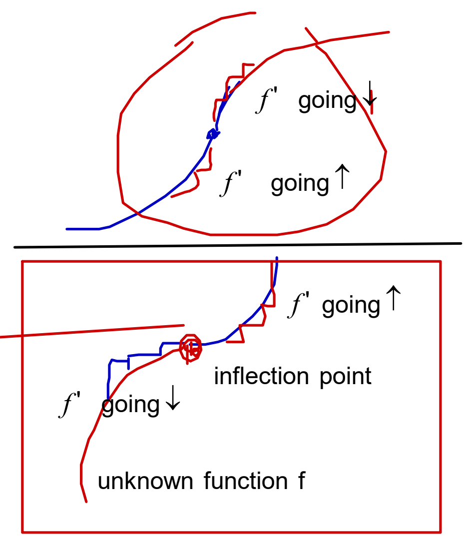


Graph of g

inflection point exxamples



. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

increasing on (0,1) and (4,6) because f'(x)=g(x)>0 f'(x)=g(x) is increasing

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

function value=initial value+accumulated change

$$f(-5)=f(1)+\int_1^{-5} g(x)\,dx = f(1)-\int_{-5}^{1} g(x)\,dx = 3-(-9-1.5+1) = \boxed{\frac{25}{2}}$$

answer

(b) Evaluate $\int_1^6 g(x)\,dx$.

$$= \underbrace{\int_1^3 g(x)\,dx}_{4} + \underbrace{\int_3^6 g(x)\,dx}_{} =$$

$$\int_3^6 2(x-4)^2\,dx = 2\int_{-1}^{2} u^2\,du = \cdot 2\left(\frac{2^3}{3}-\frac{(-1)^3}{3}\right) = 2\left(\frac{8}{3}+\frac{1}{3}\right) = 2(3) = 6$$

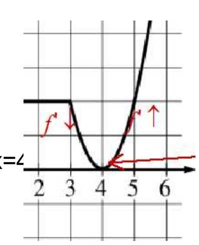
$u = x-4 \qquad : x=6 : u=2$

$du = dx \qquad : x=3 : u=-1$

final answer=4+6=$\boxed{10}$ *answer*

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

graph of f has a point of inflection at x=4 because f'(x)=g(x) changes from decreasing to increasging at x=4



question 4 from the 2018 test

2018 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

6 is not known

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

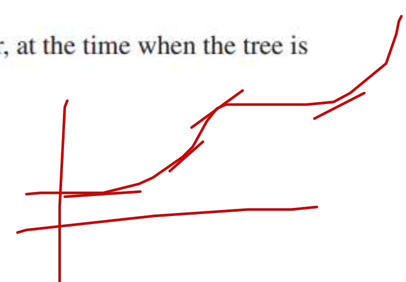
4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

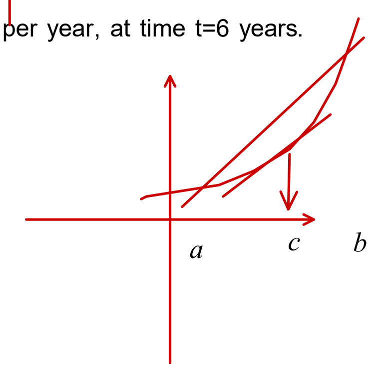


a. $H'(6) = \dfrac{H(7)-H(5)}{7-5} = \dfrac{11-6}{2} = \dfrac{5}{2}$

$H'(6) =$ is the rate at which the height of the tree is chanigng, in meters per year, at time t=6 years.

b. H'(t)=2 $\quad H$ diffrentiable $\Rightarrow continous$

$\dfrac{H(5)-H(3)}{5-3} = 2 = H'(c)$

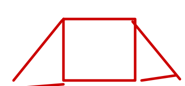


So there is some c, 3<c<5, such that $H'(c) = 2$

c. approximate the average height of the tree from t=2 to t=10

$$H_{av} = \underbrace{\frac{1}{10-2}\int_2^{10} H(t)\,dt}_{\text{average value of the function H}} \approx \frac{1}{8}\left(\frac{1.5+2}{2}\cdot 1 + \frac{2+6}{2}\cdot 2 + \frac{6+11}{2}\cdot 2 + \frac{11+15}{2}\cdot 3\right) = 263/32$$

trapezoid rule:



$area = \dfrac{b_1+b_2}{2}h$

The average height of the tree over the time interval $2 \le t \le 10$ 263/32 meters.

d. $G(x)=\frac{100x}{1+x}$ x is diameter of base of tree $G(x(t))=\frac{100(x(t))}{1+x(t)}$ $\frac{loDhi-hiDlo}{lo^2}$

$\frac{dx}{dt}=0.03$

$G(x)=50$ meters at some time t

$G=$ height, x=diameter $\Rightarrow$ both are functions of time

$G(x)=50 \quad \Rightarrow \frac{100x}{1+x}=\frac{50}{1} \Rightarrow 50+50x=100x \Rightarrow 50=100x-50x \Rightarrow 50=50x \Rightarrow 1=x$ diameter

$\frac{d}{dt}(G(x))=\frac{d}{dx}(G(x))\frac{dx}{dt}=\frac{(1+x)\cdot 100-100x(1)}{(1+x)^2}\frac{dx}{dt}=\frac{100+100x-100x}{(1+x)^2}\frac{dx}{dt}=\frac{100}{(1+x)^2}\frac{dx}{dt}$

plug in x=1 and $\frac{dx}{dt}=0.03$: $\frac{100}{(1+1)^2}\cdot\frac{3}{100}=\frac{3}{2^2}=\frac{3}{4}$

the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is 3/4 meter per year