Let G be a region and $f \in \theta(G)$ st f(z)g(z)=0 $\forall$ z $\in$ G.
Show that either f(z) $\equiv$ 0 or g(z) $\equiv$ 0 (related to identity theorem)

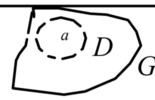
Spse f not identically 0. i.e.: f(z) $\neq$ 0 $\forall$ z $\in$ G
Then $\exists$ a nbd B(a, δ): $\{z : |z-a| < \delta\}$ s.t. f(z) $\neq$ 0 $\forall$ z $\in$ B
[because f is continous]
But f(z)g(z)=0 $\forall$ z $\in$ B $\Rightarrow g(z) = 0$ $\forall$ z $\in$ B(a, δ)
It follows by the identity theorem that g(z)=0 $\forall$ z $\in$ G.

Big Idea: Knowing something about a holomorphic function/(s) on a small disk often determines the behavior of of the function/(s) over a larger set G.

cor 1: If f $\in$ θ (G) , and $\{z \in G : f(z) = 0\}$ has a limit pt in G, then f $\equiv$ 0 i.e.f=0 $\forall$ z $\in$ G.

cor 2: (Identity Theorem) If f $\in$ θ (G) ; a $\in$ G and $\exists$ D(a,r) $\subseteq$ G where D(a,r)= $\{z : |z-a| < r\}$ s.t. f(z)=0 $\forall$ z $\in$ D(a,r). Then f(z) $\equiv$ 0.

cor 3: If f $\in$ θ (G): a $\in$ G and $\exists$ $D(a, r) = \{z : |z-a| < r\} \subseteq G$ s.t. f(z)=c $\forall$ z $\in$ D(a,r), then f(z)= c for $\forall$ z $\in$ G

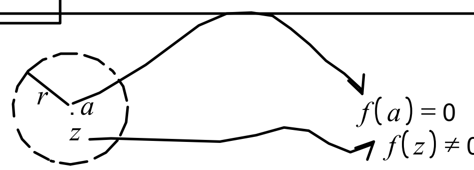


If f(a)=c, and f(D)=c, then f(G)=c

cor 4: If f $\in \theta(G)$ and g $\in$ θ (G), then f $\equiv$ g iff $\bar{z}$: $\{z \in G : f(z) = g(z)\}$ has a limit point in G. Define ϕ (z)=f(z)-g(z) and apply cor 1 above.

cor 5: If f,g $\in \theta(G)$ and $\exists$ a $\in$ G and $r > 0$ s.t. $D(a, r) \subseteq G$ and $f(z) = g(z)$ $\forall$ z $\in$ D(a,r), then f(z)=g(z) $\forall$ z $\in$ G.

Cor 6: Spse f is a nonconstant holomorphic function in G and $\exists$ a $\in$ G s.t. f(a)=0. Then $\exists$ R>0 s.t. D(a,R) $\subseteq$ G and f(z) $\neq$ 0 $\forall$ $0 < |z-a| < R$
The zeros of a nonconstant holomorphic function are isolated.



Cor 7: Spse f $\in$ θ(G) and f is not identically 0 in G.
If f(a)=0, $\exists$ an integer $n \geq 1$ and a holomorphic function g $\in$ θ (G) s.t. g(a) $\neq$ 0 and f(z)=(z-a $)^n$ g(z). $\forall$ z $\in$ G.
note that f(a)=(a-a $)^n$ g(a)=0^n g(z)=0
So we say that f has a zero of order n at a.
$f(z) = (z-a)^n g(z)$ $\forall z \in |z-a| < r$