$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

ratio test

$$\left| \frac{3^{n+1} (x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n (x+4)^n} \right|$$

$$\left| \frac{3^n \cdot 3^1 (x+4)^n \cdot (x+4)^1}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n \cdot (x+4)^n} \right|$$

$$\left| \frac{3^n \cdot 3^1 (x+4)^n \cdot (x+4)^1}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n \cdot (x+4)^n} \right|$$

$$|3(x+4)| \lim_{n\to\infty} \left(\sqrt{\frac{n}{n+1}} \right) = |3(x+4)| \cdot 1$$

$\sqrt{\frac{n}{n+1}} = \sqrt{\frac{n}{n}}$ = limit as n $\to \infty = 1$

$|3(x+4)| < 1$

$3|x+4| < 1$

$|x+4| < \frac{1}{3} \Rightarrow$ solve inequality to get interval: $-\frac{1}{3} < x+4 < \frac{1}{3}$

radius of convergence is $\frac{1}{3}$

$\boxed{\frac{-1}{3} - 4} < x < \boxed{\frac{1}{3} - 4}$

$$\frac{-1}{3} - 4 = \frac{-1}{3} - \frac{12}{3} = \frac{-13}{3}$$

$$\frac{1}{3} - 4 = \frac{1}{3} - \frac{12}{3} = \frac{-11}{3}$$

now check with x= $\frac{-13}{3}$

$$3^n \left(\frac{-13}{3} + 4 \right)^n / \sqrt{n}$$

$$\frac{-13}{3} + \frac{12}{3} = \frac{-1}{3}$$

$$\frac{3^n \left(\frac{-1}{3} \right)^n}{\sqrt{n}} = \frac{3^n \frac{(-1)^n}{3^n}}{\sqrt{n}} = \frac{(-1)^n}{\sqrt{n}} \to Converges$$

by the AST

check with x= $\frac{-11}{3}$

$$3^n \left(\frac{-11}{3} + 4 \right)^n / \sqrt{n}$$

$$-\frac{11}{3} + 4 = \frac{-11}{3} + \frac{12}{3} = \frac{1}{3}$$

$$\frac{3^n \left(\frac{1}{3} \right)^n}{\sqrt{n}} = \frac{3^n \cdot \frac{1}{3^n}}{\sqrt{n}} = \frac{1}{\sqrt{n}} \to Diverges$$

interval of convergence is :
[-13/3,-11/3)

$$\frac{(2x-1)^n}{5^n \sqrt{n}}$$

$$\left| \frac{(2x-1)^{n+1}}{5^{n+1}(\sqrt{n+1})} \cdot \frac{5^n(\sqrt{n})}{(2x-1)^n} \right| = \left| \frac{(2x-1)^n \cdot (2x-1)}{5^n \cdot 5^1 (\sqrt{n+1})} \cdot \frac{5^n(\sqrt{n})}{(2x-1)^n} \right| = \left| \frac{(2x-1)^n \cdot (2x-1)}{5^n \cdot 5^1 (\sqrt{n+1})} \cdot \frac{5^n(\sqrt{n})}{(2x-1)^n} \right|$$

$$= \left| \frac{(2x-1)\sqrt{n}}{5\sqrt{n+1}} \right|$$

$$\left| \frac{2x-1}{5} \right| \lim_{n\to\infty} \left(\sqrt{\frac{n}{n+1}} \right)$$

$$\left| \frac{2x-1}{5} \right| < 1$$

$$|2x-1| < 5$$

$$\left| 2\left(x - \frac{1}{2} \right) \right| < 5$$

$$\left| x - \frac{1}{2} \right| < \frac{5}{2}$$

$$R = \frac{5}{2}$$

$$\left| x - \frac{1}{2} \right| < \frac{5}{2}$$

$$-\frac{5}{2} < x - \frac{1}{2} < \frac{5}{2}$$

$$\frac{-5}{2} + \frac{1}{2} < x < \frac{5}{2} + \frac{1}{2}$$

$$\frac{-4}{2} < x < \frac{6}{2}$$

$$-2 < x < 3$$

when x=-2:

$$\frac{(2(-2)-1)^n}{5^n \sqrt{n}} = \frac{(-5)^n}{5^n \sqrt{n}} = \frac{(-1)^n \cdot 5^n}{5^n \sqrt{n}} = \frac{(-1)^n}{\sqrt{n}}$$

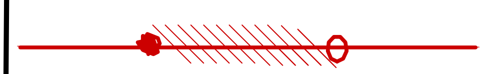
$(-1)^n$ causes a sign change
so it converges at x=-2 by the AST

when x=3

$$\frac{(2(3)-1)^n}{5^n \sqrt{n}} = \frac{5^n}{5^n \sqrt{n}} = \frac{1}{\sqrt{n}}$$

so it diverges at x=3

$I =$ [-2,3)



$$a_n = \frac{(x-2)^n}{(n^2+1)}$$

ratio test:

$$\left| \frac{(x-2)^{n+1}}{(n+1)^2+1} \cdot \frac{(n^2+1)}{(x-2)^n} \right| = \left| \frac{(x-2)^n \cdot (x-2)}{(n+1)^2+1} \cdot \frac{(n^2+1)}{(x-2)^n} \right| = \left| \frac{\cancel{(x-2)^n} \cdot (x-2)}{(n+1)^2+1} \cdot \frac{(n^2+1)}{\cancel{(x-2)^n}} \right| = |x-2| \lim_{n\to\infty} \frac{n^2+1}{(n+1)^2+1}$$ drop constants

$$= |x-2| \lim_{n\to\infty} \frac{n^2}{n^2}$$

$$= |x-2|$$

converges when |x-2|<1

$R =$ radius of convergence=1

solving the inequality above gives $-1 < x-2 < 1 \Leftrightarrow 1 < x < 3$

when x=1:

$$\frac{(1-2)^n}{n^2+1} = \frac{(-1)^n}{n^2+1} \Rightarrow converges$$ by AST

when x=3

$$\frac{(3-2)^n}{n^2+1} = \frac{1^n}{n^2+1} = \frac{1}{n^2+1}$$

$\frac{1}{n^2} \to converges$ p series with $p = 1/2$

$\frac{1}{n^2+1} \to converges$

Interval of convergence=[1,3]

$$a_n = \frac{x^{2n}}{n!}$$

$$\left| \frac{x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{x^{2n}} \right| = \left| \frac{x^{2n+2}}{(n+1)n!} \frac{n!}{x^{2n}} \right| = \left| \frac{x^{2n}x^2}{(n+1)n!} \frac{n!}{x^{2n}} \right| = \left| \frac{x^{2n}x^2}{(n+1)n!} \frac{n!}{x^{2n}} \right| = \left| \frac{x^2}{n+1} \right|$$

$|x^2| \lim_{n\to\infty} \frac{1}{n+1} = |x^2| \cdot 0 = 0 < 1$ regardless of the value of x you plug in.

This says that the series converges over $(-\infty, \infty)$

$$a_n = \frac{(-1)^n}{(2n-1)\cdot 2^n}(x-1)^n = \frac{(-1)^n (x-1)^n}{(2n-1)\cdot 2^n}$$

$$\left| \frac{(-1)^{n+1}(x-1)^{n+1}}{(2(n+1)-1)\cdot 2^{n+1}} \cdot \frac{(2n-1)\cdot 2^n}{(-1)^n(x-1)^n} \right| = \left| \frac{(-1)^n(-1)(x-1)^n\cdot(x-1)}{(2n+2-1)\cdot 2^n\cdot 2} \cdot \frac{(2n-1)\cdot 2^n}{(-1)^n(x-1)^n} \right| = \left| \frac{(-1)^n(-1)(x-1)^n\cdot(x-1)}{(2n+1)\cdot 2^n\cdot 2} \cdot \frac{(2n-1)\cdot 2^n}{(-1)^n(x-1)^n} \right|$$

$$= \left| \frac{(-1)^n(-1)(x-1)^n\cdot(x-1)}{(2n+1)\cdot 2^n\cdot 2} \cdot \frac{(2n-1)\cdot 2^n}{(-1)^n(x-1)^n} \right| = \left| \frac{-1(x-1)}{(2n+1)\cdot 2}(2n-1) \right| = \left| \frac{-1(x-1)}{2} \right| \lim_{n\to\infty} \frac{2n-1}{2n+1}$$

R=2

$$\left| \frac{-1(x-1)}{2} \right| < 1 \Leftrightarrow \left| \frac{x-1}{2} \right| < 1 \Leftrightarrow |x-1| < 2 \Leftrightarrow -2 < x-1 < 2 \Leftrightarrow -1 < x < 3$$

when x=-1:

$$\frac{(-1)^n}{(2n-1)\cdot 2^n}(-1-1)^n = \frac{(-1)^n}{(2n-1)\cdot 2^n}(-2)^n = \frac{(-1)^n}{(2n-1)\cdot 2^n}(-1)^n\cdot 2^n = \frac{(-1)^n\cdot(-1)^n}{2n-1} = \frac{1}{2n-1}$$

By the Limit Comparison Test, $\frac{1}{2n-1}$ diverges. $b_n = \frac{1}{n}$

when x=3

$$\frac{(-1)^n}{(2n-1)\cdot 2^n}(3-1)^n = \frac{(-1)^n}{(2n-1)\cdot 2^n}\cdot 2^n = \frac{(-1)^n}{(2n-1)\cdot 2^n}\cdot 2^n = \frac{(-1)^n}{2n-1}$$ $\Rightarrow$ converges by the AST

Interval of Convergence: (-1,3]

$$a_n = \frac{x^n}{n^4 \cdot 4^n}$$

$$\left| \frac{x^{n+1}}{(n+1)^4 \cdot 4^{n+1}} \cdot \frac{n^4 \cdot 4^n}{x^n} \right| = \left| \frac{x^n x^1}{(n+1)^4 \cdot 4^n \cdot 4^1} \cdot \frac{n^4 \cdot 4^n}{x^n} \right| = \left| \frac{x^n x^1}{(n+1)^4 \cdot 4^n \cdot 4^1} \cdot \frac{n^4 \cdot 4^n}{x^n} \right| = \left| \frac{x^1}{(n+1)^4 \cdot 4} n^4 \right| = \left| \frac{x}{4} \right| \lim_{n \to \infty} \frac{n^4}{(n+1)^4}$$

$$= \left| \frac{x}{4} \right| \lim_{n \to \infty} \frac{n^4}{(n)^4}$$

$$= \left| \frac{x}{4} \right|$$

converges when $\left| \frac{x}{4} \right| < 1 \Leftrightarrow |x| < 4 \Leftrightarrow -4 < x < 4$

$\mathbf{R} = 4$

check with x=-4:

$$\frac{(-4)^n}{n^4 \cdot 4^n} = \frac{(-1)^n \cdot 4^n}{n^4 \cdot 4^n} = \frac{(-1)^n}{n^4} \Rightarrow converges$$ by AST

check with x=4:

$$\frac{4^n}{n^4 \cdot 4^n} = \frac{1}{n^4}$$ p series with p=4>1, so it converges also at x=4

So the interval of convergence is [-4,4]